2.8.1 Procedure for Analysis

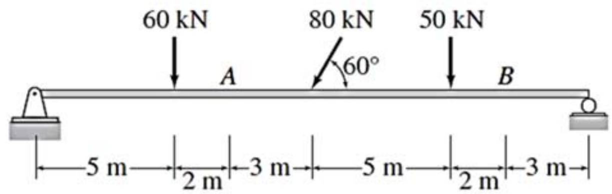
The procedure for determining internal forces at a specified location on a beam can be summarized as follows:

1- Compute the support reactions by applying the equations of equilibrium and condition (if any) to the free body of the entire beam. In cantilever beams, this step can be avoided by selecting the free, or externally unsupported, portion of the beam for analysis.
2- Pass a section perpendicular to the centroidal axis of the beam at the point where the internal forces are desired, thereby cutting the beam into two portions.
3- Although either of the two portions of the beam can be used for computing internal forces, we should select the portion that will require the least amount of computational effort, such as the portion that does not have any reactions acting on it or that has the least number of external loads and reactions applied to it.
4- Determine the axial force at the section by algebraically summing the components in the direction parallel to the axis of the beam of all the external loads and support reactions acting on the selected portion.
5- Determine the shear at the section by algebraically summing the components in the direction perpendicular to the axis of the beam of all the external loads and reactions acting on the selected portion.
6- Determine the bending moment at the section by algebraically summing the moments about the section of all the external forces plus the moments of any external couples acting on the selected portion.
7- To check the calculations, values of some or all of the internal forces may be computed by using the portion of the beam not utilized in steps 4 through 6. If the analysis has been performed correctly, then the results based on both left and right portions must be identical.

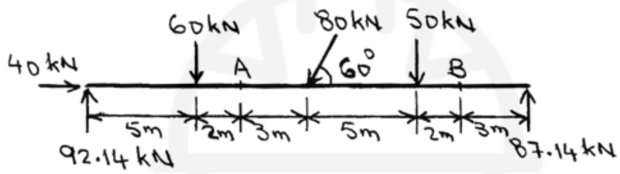
For the following examples, determine the axial forces, shears, and bending moments at points A and B of the structure shown.

2.8.2 Examples:

Example (1):



Solution:



$Q_A = \underline{-40\ \text{kN}}$

$S_A = 92.14 - 60 = \underline{32.14\ \text{kN}}$

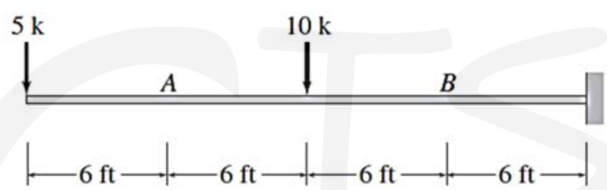
$M_A = 92.14(7) - 60(2) = \underline{524.98\ \text{kN.m}}$

$Q_B = \underline{0}$

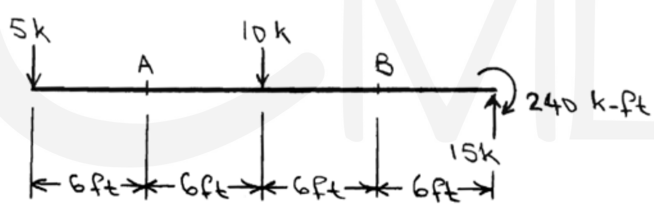
$S_B = \underline{-87.14\ \text{kN}}$

$M_B = 87.14(3) = \underline{261.42\ \text{kN.m}}$

Example (2):



Solution:



$Q_A = \underline{0}$

$S_A = \underline{-5\ \text{k}}$

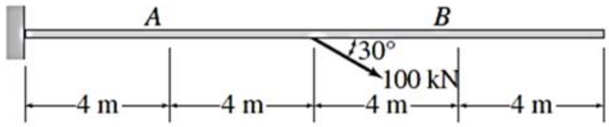
$M_A = -5(6) = \underline{-30\ \text{k-ft}}$

$Q_B = \underline{0}$

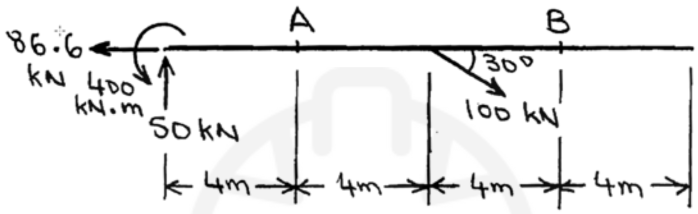
$S_B = -5 - 10 = \underline{-15\ \text{k}}$

$M_B = -5(18) - 10(6) = \underline{-150\ \text{k-ft}}$

Example (3):



Solution:



$Q_A = 100 \cos 30^\circ = \underline{86.6 \text{ kN}}$

$S_A = 100 \sin 30^\circ = \underline{50 \text{ kN}}$

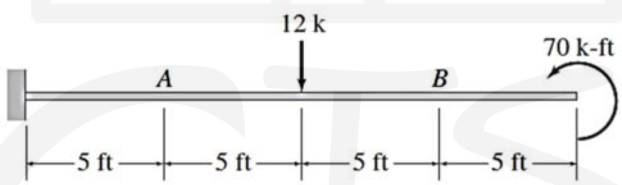
$M_A = -100 \sin 30^\circ (4) = \underline{-200 \text{ kN.m}}$

$Q_B = \underline{0}$

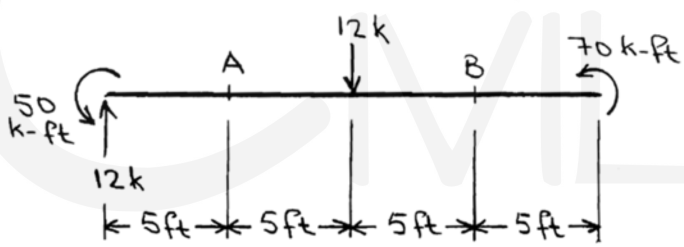
$S_B = \underline{0}$

$M_B = \underline{0}$

Example (4):



Solution:



$Q_A = \underline{0}$

$S_A = \underline{12 \text{ k}}$

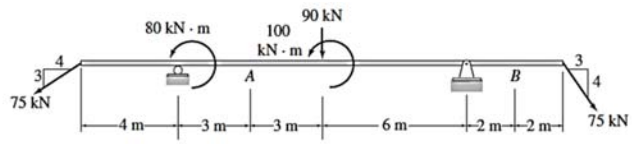
$M_A = 12(5) - 50 = \underline{10 \text{ k-ft}}$

$Q_B = \underline{0}$

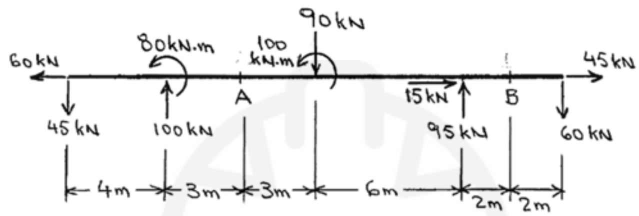
$S_B = \underline{0}$

$M_B = \underline{70 \text{ k-ft}}$

Example (5):



Solution:



$Q_A = \underline{60 \text{ kN}}$

$S_A = -45 + 100 = \underline{55 \text{ kN}}$

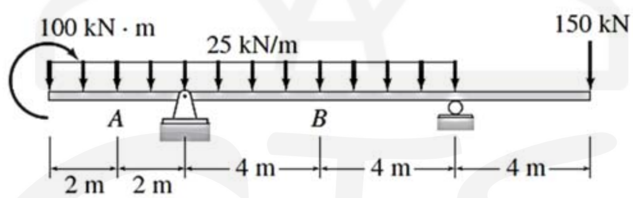
$M_A = -45(7) - 80 + 100(3) = \underline{-95 \text{ kN.m}}$

$Q_B = \underline{45 \text{ kN}}$

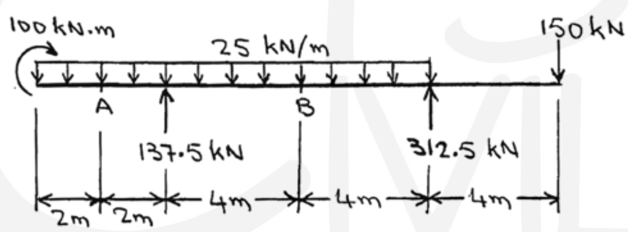
$S_B = \underline{60 \text{ kN}}$

$M_B = -60(2) = \underline{-120 \text{ kN.m}}$

Example (6):



Solution:



$Q_A = \underline{0}$

$S_A = -25(2) = \underline{-50 \text{ kN}}$

$M_A = 100 - 25(2)(1) = \underline{50 \text{ kN.m}}$

$Q_B = \underline{0}$

$S_B = 150 - 312.5 + 25(4) = \underline{-62.5 \text{ kN}}$

$M_B = -150(8) + 312.5(4) - 25(4)(2)$

$M_B = \underline{-150 \text{ kN.m}}$

2.9 Shear Force and Bending Moment Diagrams:

2.9.1 Procedure for Analysis

The following step-by-step procedure can be used for constructing the shear and bending moment diagrams for beams by applying the foregoing relationships between the loads, the shears, and the bending moments.

1- Calculate the support reactions.

2- Construct the shear diagram as follows:

a. Determine the shear at the left end of the beam. If no concentrated load is applied at this point, the shear is zero at this point; go to step 2(b). Otherwise, the ordinate of the shear diagram at this point changes abruptly from zero to the magnitude of the concentrated force. Recall that an upward force causes the shear to increase, whereas a downward force causes the shear to decrease.

b. Proceeding from the point at which the shear was computed in the previous step toward the right along the length of the beam, identify the next point at which the numerical value of the ordinate of the shear diagram is to be determined. Usually, it is necessary to determine such values only at the ends of the beam and at points at which the concentrated forces are applied and where the load distributions change.

c. Determine the ordinate of the shear diagram at the point selected in step 2(b) (or just to the left of it, if a concentrated load acts at the point) by adding algebraically the area under the load diagram between the previous point and the point currently under consideration to the shear at the previous point (or just to the right of it, if a concentrated force act at the point).

d. Determine the shape of the shear diagram between the previous point and the point currently under consideration, (that the slope of the shear diagram at a point is equal to the load intensity at that point).

e. If no concentrated force is acting at the point under consideration, then proceed to step 2(f). Otherwise, determine the ordinate of the shear diagram just to the right of the point by adding algebraically the magnitude of the concentrated load to the shear just to the left of the point. Thus, the shear diagram at this point changes abruptly by an amount equal to the magnitude of the concentrated force.

f. If the point under consideration is not located at the right end of the beam, then return to step 2(b). Otherwise, the shear diagram has been completed. If the analysis has been carried out correctly, then the value of shear just to the right of the right end of the beam must be zero, except for the round-off errors.

3- Construct the bending moment diagram as follows:

a. Determine the bending moment at the left end of the beam. If no couple is applied at this point, the bending moment is zero at this point; go to step 3(b). Otherwise, the ordinate of the bending moment diagram at this point changes abruptly from zero to the magnitude of the moment of the couple. Recall that a clockwise couple causes the bending moment to increase, whereas a counterclockwise couple causes the bending moment to decrease at its point of application.

b. Proceeding from the point at which the bending moment was computed in the previous step toward the right along the length of the beam, identify the next point at which the numerical value of the ordinate of the bending moment diagram is to be determined. It is usually necessary to determine such values only at the points where the numerical values of shear were computed in step 2, where the couples are applied, and where the maximum and minimum values of bending moment occur. In addition to the points of application of couples, the maximum and minimum values of bending moment occur at points where the shear is zero. At a point of zero shear, if the shear changes from positive to the left to negative to the right, the slope of the bending moment diagram will change from positive to the left of the point to negative to the right of it; that is, the bending moment will be maximum at this point. Conversely, at a point of zero shear, where the shear changes from negative to the left to positive to the right, the bending moment will be minimum. For most common loading conditions, such as concentrated loads and uniformly and linearly distributed loads, the points of zero shear can be located by considering the geometry of the shear diagram. However, for some cases of linearly distributed loads, as well as for nonlinearly distributed loads, it becomes necessary to locate the points of zero shear by solving the expressions for shear.

c. Determine the ordinate of the bending moment diagram at the point selected in step 3(b) (or just to the left of it, if a couple acts at the point) by adding algebraically the area under the shear diagram between the previous point and the point currently under consideration to the bending moment at the previous point (or just to the right of it, if a couple acts at the point).

d. Determine the shape of the bending moment diagram between the previous point and the point currently under consideration (the slope of the bending moment diagram at a point is equal to the shear at that point).

e. If no couple is acting at the point under consideration, then proceed to step 3(f). Otherwise, determine the ordinate of the bending moment diagram just to the right of the point by adding algebraically the magnitude of the moment of the couple to the bending moment just to the left of the point. Thus, the bending moment diagram at this point changes abruptly by an amount equal to the magnitude of the moment of the couple.

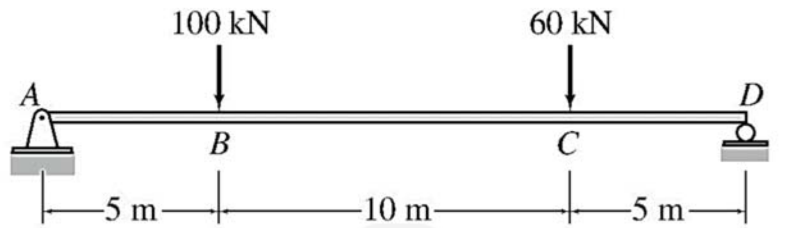
f. If the point under consideration is not located at the right end of the beam, then return to step 3(b). Otherwise, the bending moment diagram has been completed. If the analysis has been carried out correctly, then the value of bending moment just to the right of the right end of the beam must be zero, except for the round-off errors.

The foregoing procedure can be used for constructing the shear and bending moment diagrams by proceeding from the left end of the beam to its right end, as is currently the common practice. However, if we wish to construct these diagrams by proceeding from the right end of the beam toward the left, the procedure essentially remains the same except that downward forces must now be considered to cause increase in shear, counterclockwise couples are now considered to cause increase in bending moment, and vice versa.

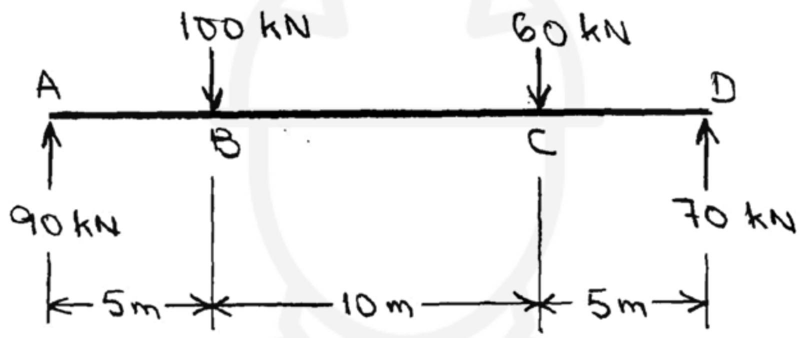
For the following examples, draw the shear and bending moment diagrams and the qualitative deflected shape for the beam shown.

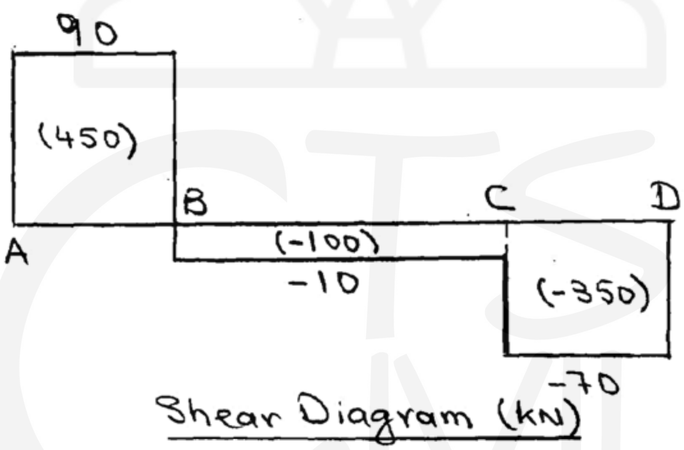
2.9.2 Examples:

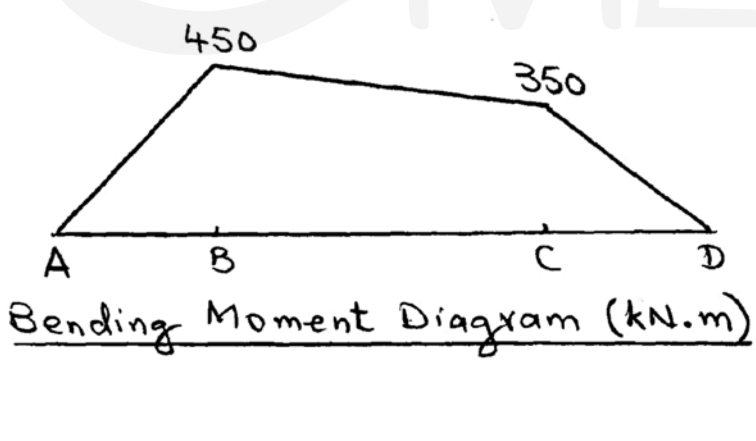
Example (1):



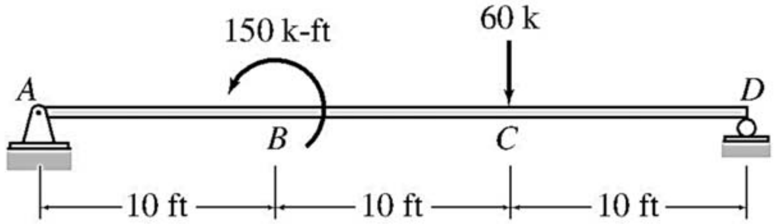
Solution:



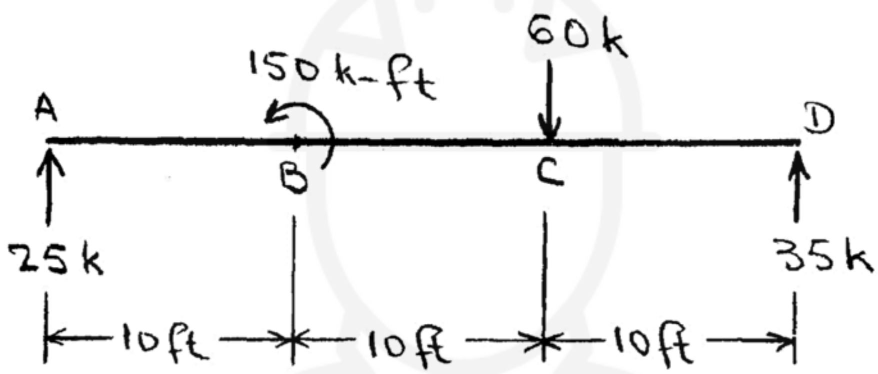


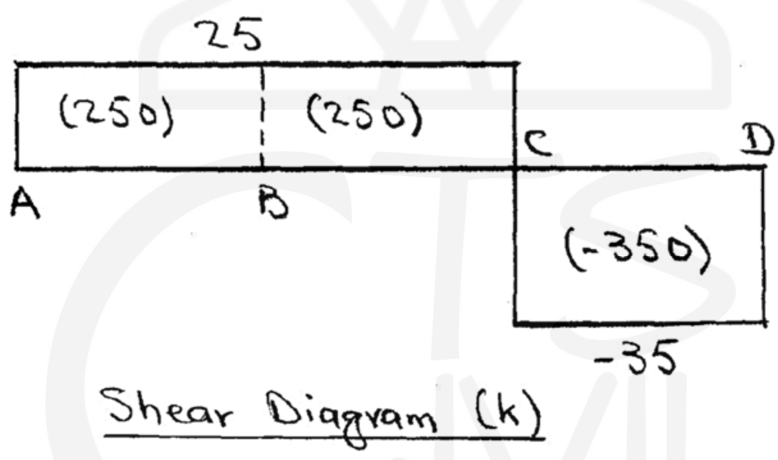


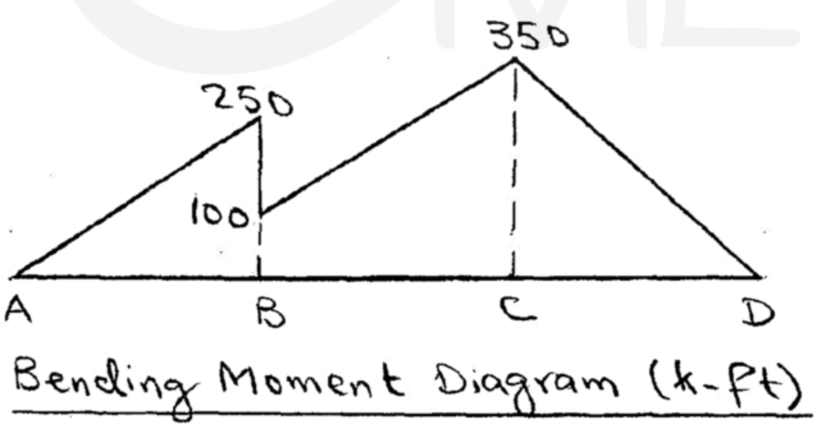
Example (2):



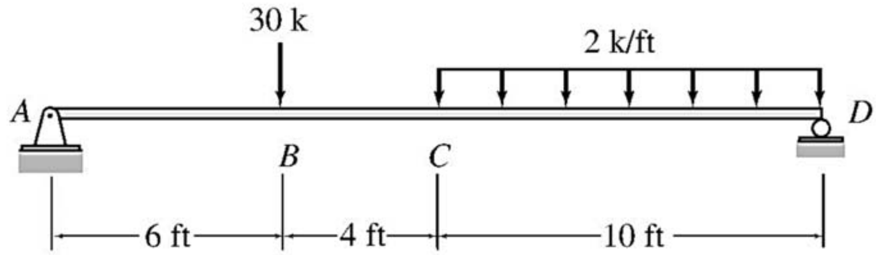
Solution:



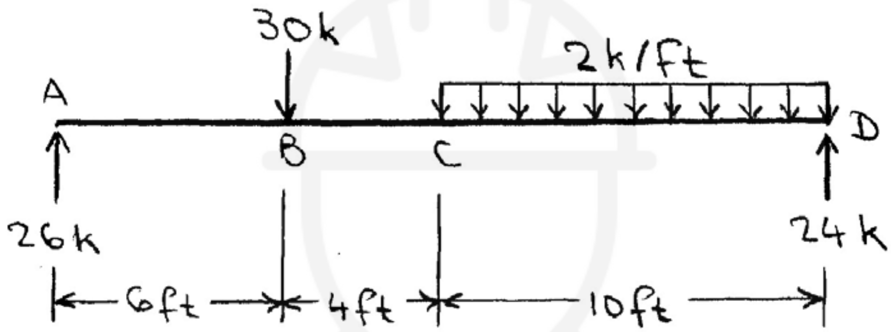


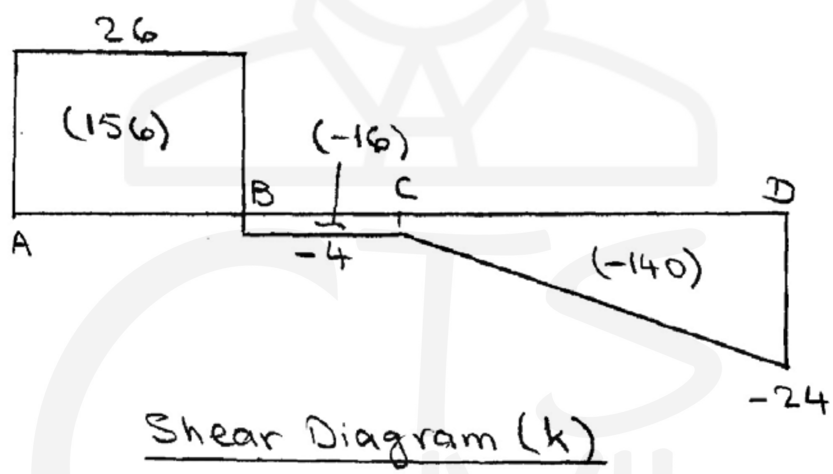


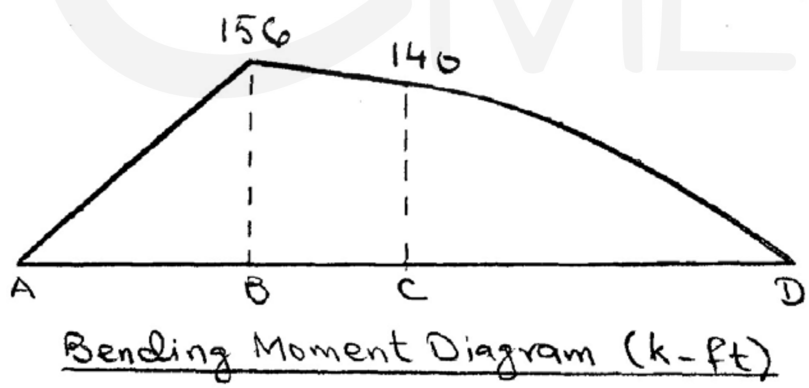
Example (3):



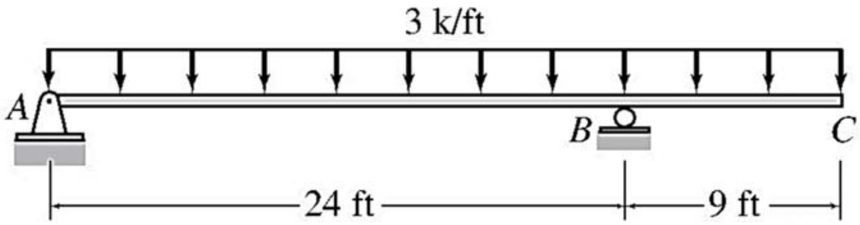
Solution:



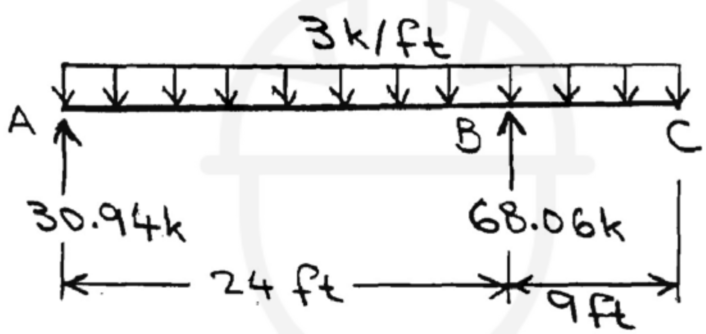


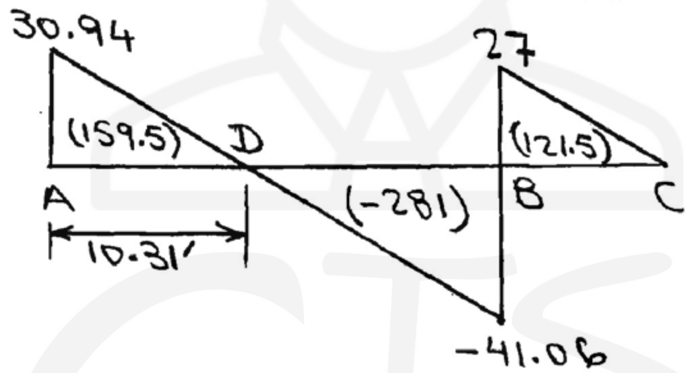


Example (4):

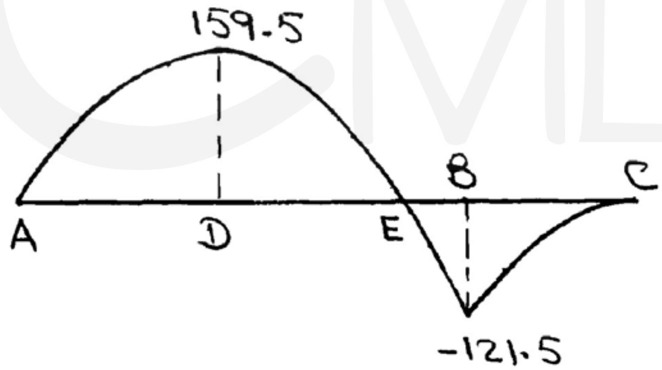


Solution:



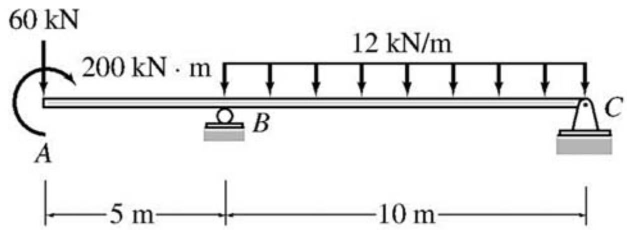


Shear Diagram (k)

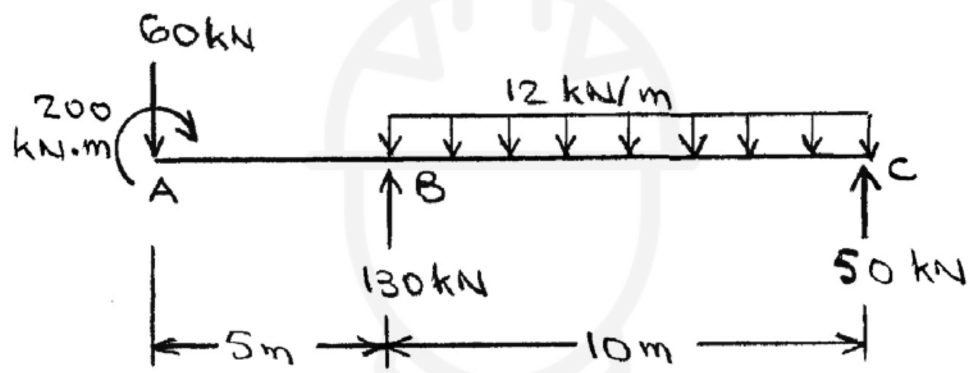


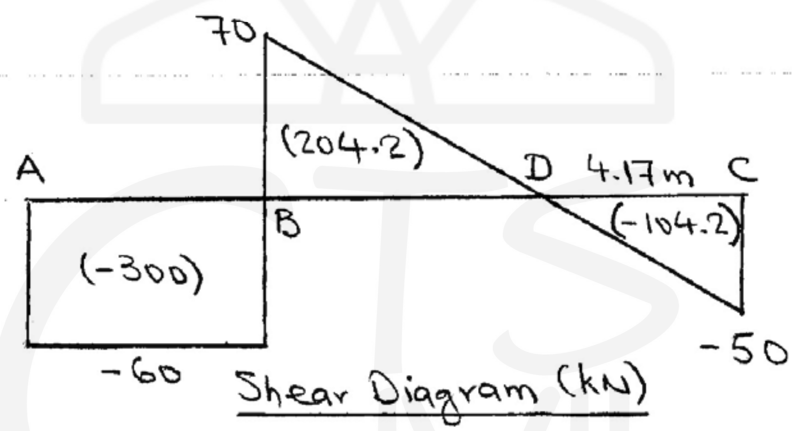
Bending Moment Diagram (k-ft)

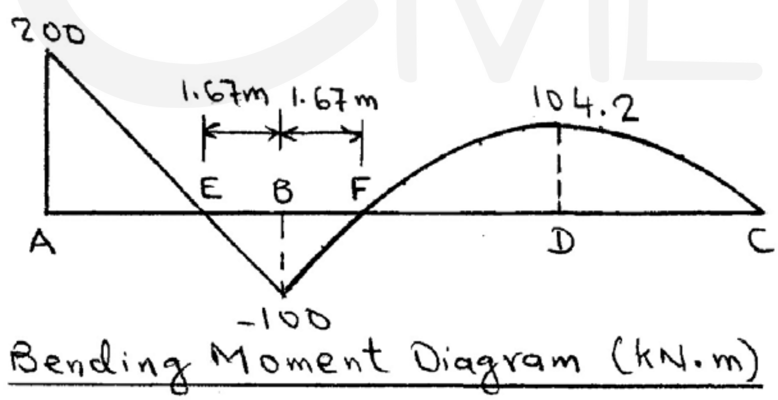
Example (5):



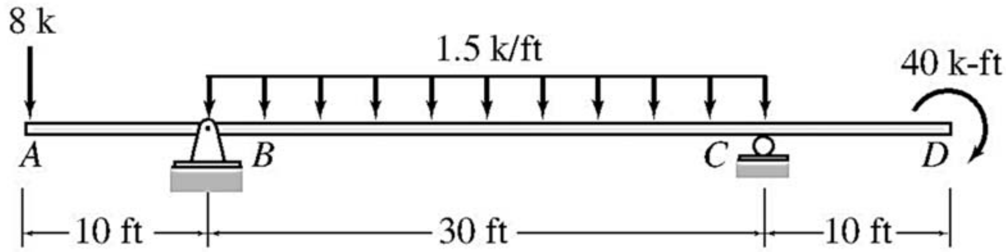
Solution:



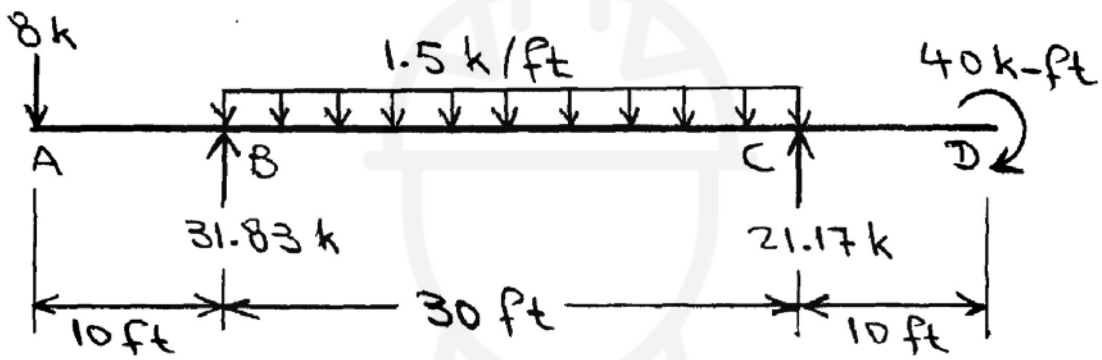


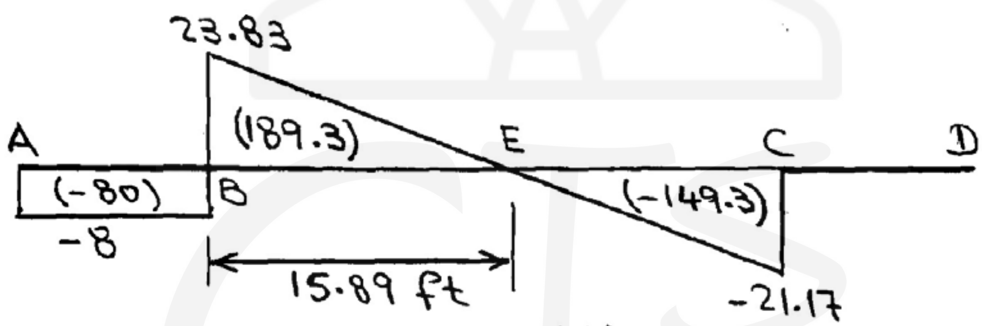


Example (6):

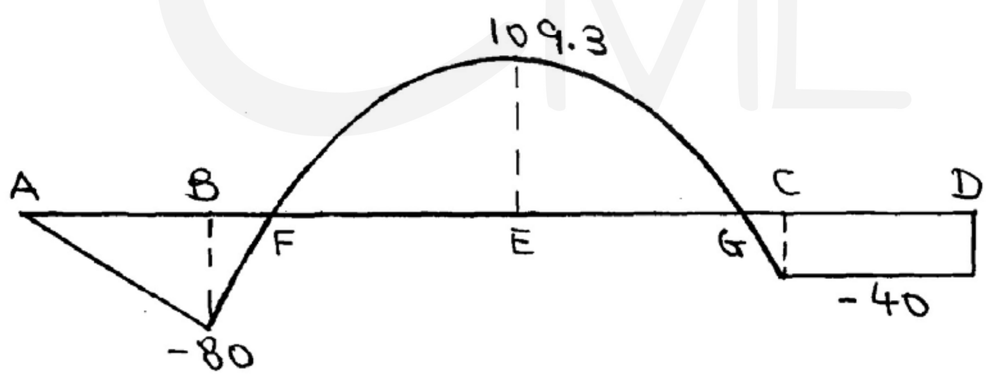


Solution:



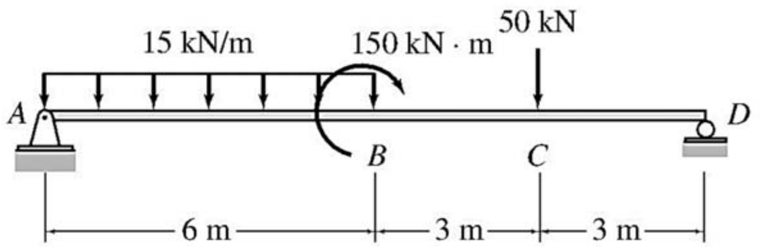


Shear Diagram (k)

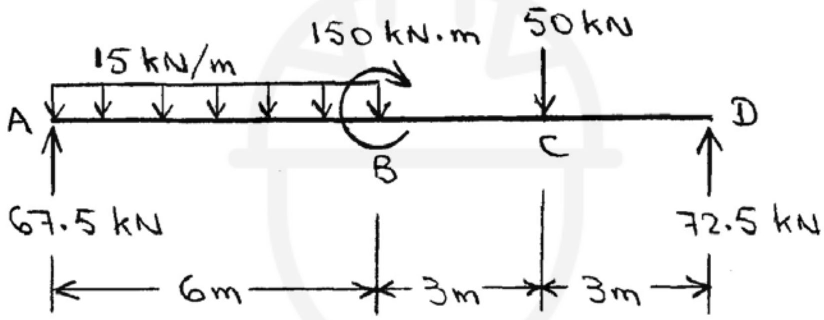


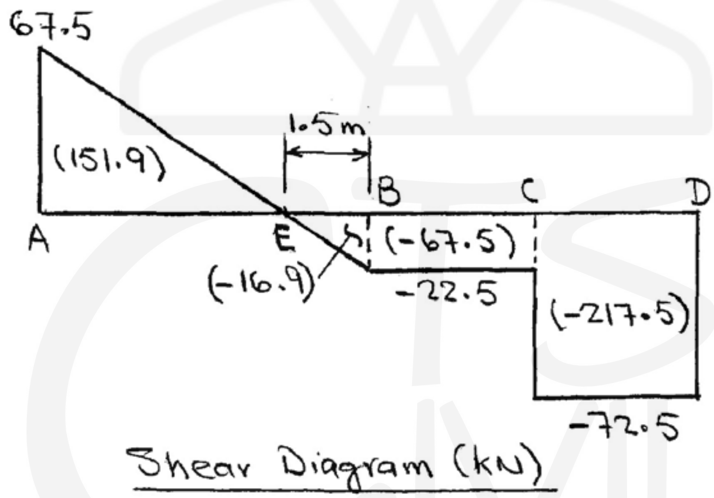
Bending Moment Diagram (k-ft)

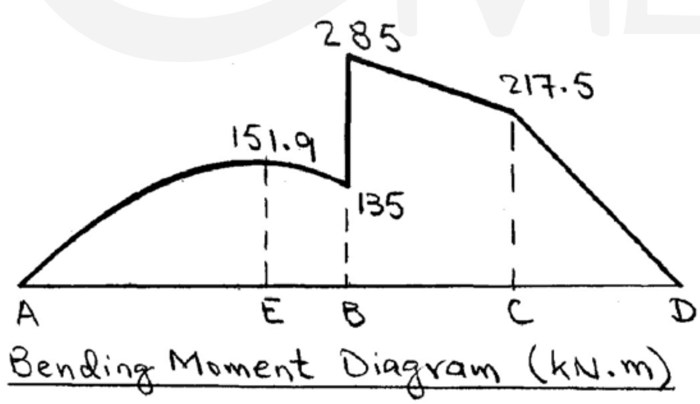
Example (7):



Solution:







2.10 Problems:

Question № 1:
For the beam shown in figure (1), What are the values of the shear force and bending moment at $x = \frac{l}{2}$?

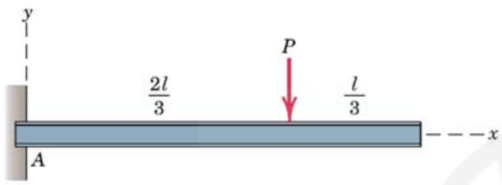


Figure 1

Question № 2:
For the beam shown in figure (2), determine the shear force V at a section B between A and C and the moment M at the support A.

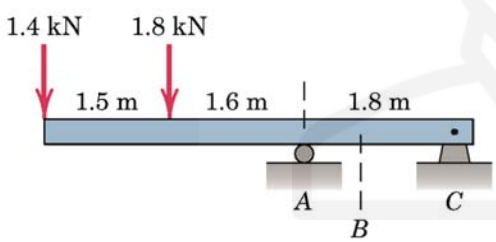


Figure 2

Question № 3:
Determine the shear V and bending moment M at a section of the loaded beam shown in figure (3) 200 mm to the right of A.

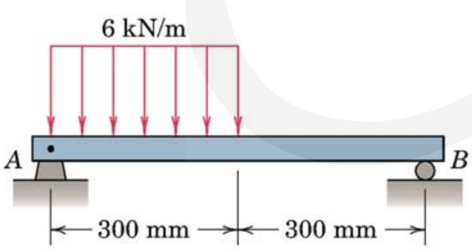


Figure 3

Question № 4:
Determine the shear V and bending moment M at a section of the loaded beam shown in figure (4) 2 m to the right of support A.

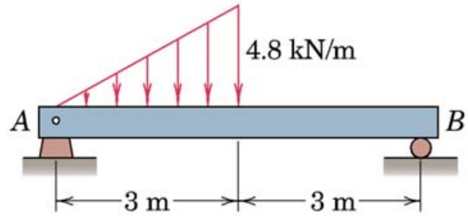


Figure 4

Question № 5:
For the beam shown in figure (5), find the shear force and bending moment at points C and D.

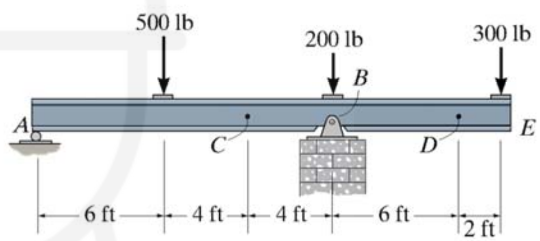


Figure 5

Question № 6:
For the beam shown in figure (6), find the shear force and bending moment at point C. Assume support A is a hinge and B is a roller.

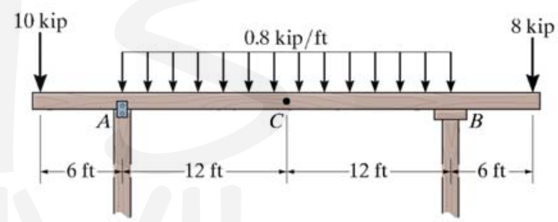


Figure 6

Question № 7:
For the beam shown in figure (7), What is the shear force and bending moment at a distance $(\frac{L}{2})$ from the left support? Assume support A is a hinge and B is a roller.

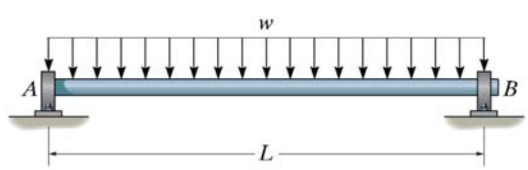


Figure 7

Question № 8:
For the beam shown in figure (8):

- Draw the shear force and bending moment diagrams.
- What are the values of the shear force and bending moment at points C and D.

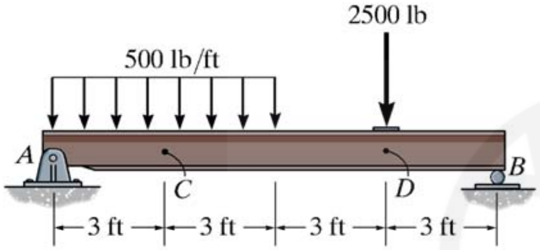


Figure 8

Question № 9:
For the beam shown in figure (9), draw the shear force and bending moment diagram. Assume support A is a hinge and B is a roller.

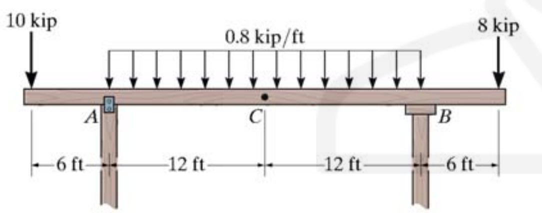


Figure 9

Question № 10:
For the beam shown in figure (10):

- Draw the shear force and bending moment diagrams.
- At what distance from support A the moment is zero?

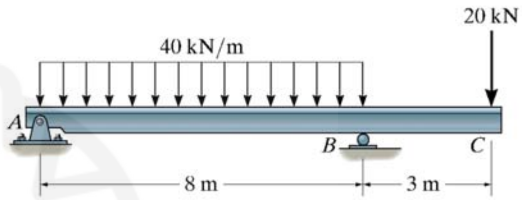


Figure 10

Question № 11:
Draw the shear force and bending moment diagram for the cantilever beam shown in figure (11).

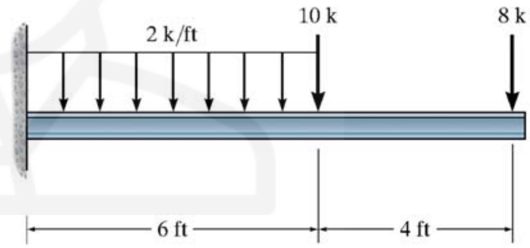


Figure 11